

Real Options Analysis of Fishing Fleet Dynamics: A Test

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Summary

This paper develops and tests a dynamic optimization model of fishermen's investment behavior in a limited-entry fishery. Because exit from limited-entry fisheries may be irreversible, the fisherman has an incentive to maintain the right to fish (whether by actually fishing or by purchasing an annual license) even when the fishery is not profitable, in the hope that conditions may improve. This incentive provides at least a partial explanation for excess capacity in fishing fleets, one of the most pressing fisheries management issues in limited-entry (and other) fisheries around the world. To assess the ability of simple financial models to explain observed investment behavior, we develop a two-factor (price and catch) real options model of the decision problem faced by an active fisherman who has the option to exit a fishery irrevocably. The immediate reason for adopting a two-factor model is the hope of achieving greater predictive power, since obviously both price and catch are important to fishermen's decisions. Another advantage to this approach is that it provides a mechanism by which investment behavior can be linked in a real options framework to exogenous factors that affect price and catch separately. For example, international market forces are likely to affect price while having a negligible effect on a local fish stock, while local fish stock dynamics may affect catch directly but have little influence on prices (assuming the demand for a particular fish is relatively elastic). In a comparison of model predictions about fishermen's exit decisions to 5059 observed decisions in the California salmon fishery in the 1990s, 65% of the model's predictions are correct, suggesting this approach may be useful in the analysis of fishing fleet dynamics.

Keywords: Real option investment, Numerical methods, Fisheries

JEL Classification: Q22, D8

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1. Introduction

Fishermen make investment decisions (e.g., to buy or sell a boat or fishing permit) in a stochastic and dynamic context, usually under considerable uncertainty about how much fish will be caught and what price it will fetch. In some cases, such as the purchase of specialized equipment, the decision may be largely irreversible. Much of the literature on fisheries investment has taken the approach of determining the socially optimal level of harvest or fleet capacity (Boyce (1995); Clark and Munro (1979)). Other studies have attempted to explain observed fleet dynamics in an aggregate framework (Bjorndal and Conrad (1987); Fletcher, Howitt, and Johnson (1988)). Relatively few studies have attempted empirical individual-based models of fisheries investments. A recent paper by Helu *et al.* (1999) represented individual boats in an aggregate simulation model, and Bockstael and Opaluch (1983) and Ward and Sutinen (1994) have applied econometric models to individual entry-exit behavior.

Tomberlin (2002) developed a model of individual fishermen's decisions to enter or exit a fishery based on stochastic finance theory as developed in the real options literature (for a good introduction to this literature, see Dixit and Pindyck (1994)). That model treated a boat's revenue as a stochastic process and derived an expression for the value of a fishing enterprise that includes the values of options conferred by managerial flexibility. For example, in California's commercial salmon fishery, an active boat has the option to exit the fishery but not to re-enter it once the permit has been surrendered, and can renew this option each year by purchasing a salmon vessel permit. For the period 1981-99, Tomberlin's model correctly predicted 55-73% of observed decisions in the California salmon fleet, depending on model specification, but there are obvious shortcomings with the model structure. The present paper aims to improve the theoretical basis of that earlier work, and presumably its predictive power, by explicitly treating fish price and quantity landings as separate processes, instead of modeling a single revenue process. The following sections describe the model, the estimation of model parameters, and an application of the model to the California salmon fishery. We find that 65% of the model's predictions are corroborated by observed behavior, and that the greatest source of error in the model's predictions is its inability to represent well the behavior of small boats that fish occasionally and can probably be thought of as an informal sector of the fishery.

2. A Model of the Exit-or-Stay Decision

Many fishermen in California target salmon almost exclusively, making it reasonable to consider salmon fishing as a project in itself, separate from other projects such as tuna fishing or alternative onshore employment. As long as a fisherman remains in the salmon fishery, he receives a profit flow

$$(1) \quad \pi(P_t, Q_t) = (P_t * Q_t - C_t - L_t),$$

where P is the price and Q the quantity of fish landed, C is the operating cost flow (which may itself be a function of Q) and L is the periodic license fee. Because salmon is a limited-entry fishery, the decision to exit the fishery is irrevocable: once the salmon vessel permit has been allowed to lapse, the fisherman cannot get it back and cannot land salmon again. In reality, it is possible for the fisherman to maintain the license while suspending fishing activity, but we ignore that possibility for the purposes of this paper, since it significantly complicates the analysis.

Suppose that price, P , and catch, Q , each follows an independent geometric Brownian motion:

$$(2) \quad dP = \alpha_p P dt + \sigma_p P dz_p$$

$$(3) \quad dQ = \alpha_q Q dt + \sigma_q Q dz_q,$$

where $\alpha_p, \sigma_p, \alpha_q, \sigma_q$ are the drift and the volatility parameters of the price and quantity processes, respectively. Assuming an exogenous discount rate ρ , we can apply the methods of stochastic dynamic programming and Ito's lemma to arrive at a partial differential equation describing the expected value of salmon fishing with an option to quit:

$$(4) \quad \frac{1}{2} \frac{\partial^2 V_1}{\partial P^2} P^2 \sigma_p^2 + \frac{1}{2} \frac{\partial^2 V_1}{\partial Q^2} Q^2 \sigma_q^2 + \alpha_p \frac{\partial V_1}{\partial P} P + \alpha_q \frac{\partial V_1}{\partial Q} Q - \rho V_1 + PQ - C - L = 0.$$

This expression can be easily modified in order to allow correlation between the processes (2) and (3). We treat them as independent processes because the price of salmon is to a large extent determined in the international market and because modeling the effect of price on landings would require a model of fishing effort, which is beyond the scope of this paper: our purpose here is simply to assess the predictive power of the simplest model, in which P and Q are independent.

If the fisherman exits the salmon fishery, he receives no current income from salmon and is not permitted any future income, so the value of the salmon fishing project is simply

$$(5) \quad V_0 = 0.$$

Exiting the fishery does, however, enable the fisherman to obtain a salvage value S , which could in many fisheries be the sale price of a boat or a transferable permit. In our case, since the project defined by V_I and V_0 is salmon fishing per se, we define S as the capitalized value of profit available to the fisherman from pursuing other fisheries in the time he would otherwise be fishing for salmon. The problem is to identify $\{P_x, Q_x\}$ combinations at which the expected value of the active project V_I is the same as the expected value of the inactive project V_0 plus the salvage value:

$$(6) \quad V_I(P_x, Q_x) = V_0(P_x, Q_x) + S.$$

This is the value-matching condition, which is complemented by the smooth-pasting conditions (see e.g. Dixit and Pindyck (1994)):

$$(7) \quad \frac{\partial V_I(P_x, Q_x)}{\partial Q} = \frac{\partial V_0(P_x, Q_x)}{\partial Q},$$

$$(8) \quad \frac{\partial V_I(P_x, Q_x)}{\partial P} = \frac{\partial V_0(P_x, Q_x)}{\partial P}.$$

which in turns means, given (5), that on the threshold frontier we must have the partial derivatives of the active fishing project with respect to both price and quantity, $\frac{\partial V_I(P_x, Q_x)}{\partial P}$ and $\frac{\partial V_I(P_x, Q_x)}{\partial Q}$, equal to zero.

The solution to the system (4-5) subject to conditions (6-8) enables us to identify the threshold set $\{P_x, Q_x\}$, below which exit is optimal and above which remaining active is optimal. Comparing a boat's realized values of P and Q to the frontier described by $\{P_x, Q_x\}$, given the maintained hypothesis of expected value maximizing behavior, provides a means for testing the explanatory power of the model.

We estimated the $\{P_x, Q_x\}$ frontier by solving the partial differential equation (4) using the finite difference method, with (6) and (7-8) as boundary conditions. Numerical solution methods, and finite difference methods in particular, are extremely efficient when dealing with two random factors models, see Wilmott *et al.* (2002) for an extensive treatment of numerical techniques. Although the model does not include time as an explicit variable in the value function, since the problem is in principle an infinite horizon problem, for the sake of numerical solution we

approximate the infinite horizon project and option values by a large value of terminal time T_{MAX} . Therefore, the equation in (4) is rewritten as:

$$(4') \quad \frac{1}{2} \frac{\partial^2 V_1}{\partial P^2} P^2 \sigma_P^2 + \frac{1}{2} \frac{\partial^2 V_1}{\partial Q^2} Q^2 \sigma_Q^2 + \alpha_P \frac{\partial V_1}{\partial P} P + \alpha_Q \frac{\partial V_1}{\partial Q} Q + \frac{\partial V_1}{\partial t} - \rho V_1 + PQ - C - L = 0.$$

The finite difference method consists of replacing the partial derivatives which occur in partial differential equations by difference equations based on Taylor series expansions. A three-dimensional mesh replaces the continuous price-quantity-time space. Each axis is divided into equally spaced points, δp , δq and δt , hence each mesh point is defined by a triplet of coordinates in the form $(i\delta p, z\delta q, j\delta t)$.

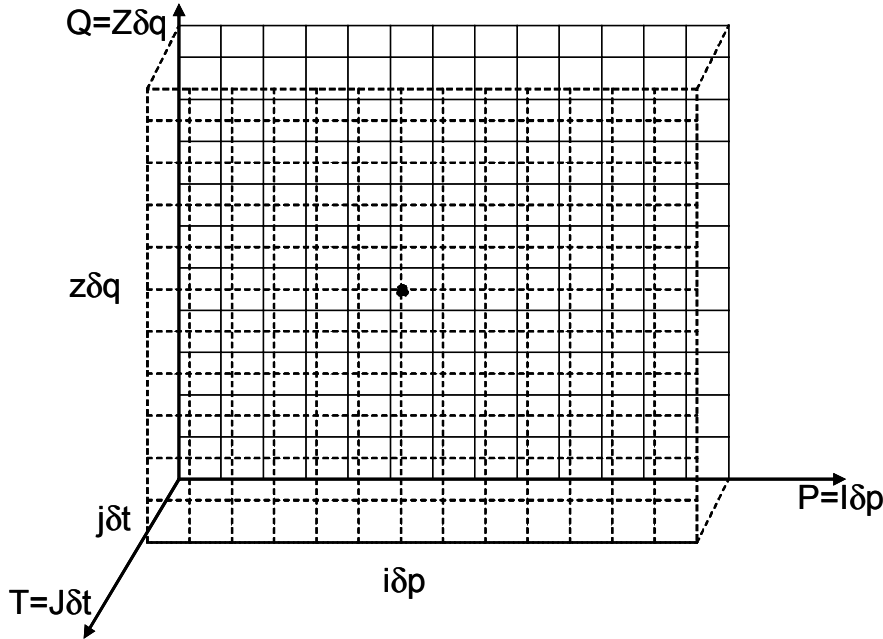


Figure 1. The finite difference mesh

The value function, $V(P,Q,T)$ needs to be calculated only at the mesh points, and, for example, the value at $(i\delta p, z\delta q, j\delta t)$ can be written as

$$V_{i,z}^j = V(i\delta p, z\delta q, j\delta t).$$

We define the first order derivative with respect to time, using the backward difference approximation, as we want to apply the implicit finite difference method¹, as:

$$(9) \quad \frac{\partial V(i\delta p, z\delta q, j\delta t)}{\partial t} \approx \frac{V_{i,z}^j - V_{i,z}^{j-1}}{\delta t} + O((\delta t)).$$

¹ Implicit finite difference methods are used to overcome stability limitations which apply to the explicit methods, see Wilmott *et al.* (2002) for a discussion of advantages and limitations of different finite difference methods.

For first order derivatives with respect to price and quantity we use central difference approximation:

$$(10) \quad \frac{\partial V(i\delta p, z\delta q, j\delta t)}{\partial P} \approx \frac{V_{i+1,z}^j - V_{i-1,z}^j}{2\delta p} + O((\delta p)),$$

$$(11) \quad \frac{\partial V(i\delta p, z\delta q, j\delta t)}{\partial Q} \approx \frac{V_{i,z+1}^j - V_{i,z-1}^j}{2\delta q} + O((\delta q)).$$

Finally, for second order derivatives, symmetric central difference approximations are used:

$$(12) \quad \frac{\partial^2 V(i\delta p, z\delta q, j\delta t)}{\partial P^2} \approx \frac{V_{i+1,z}^j - 2V_{i,z}^j + V_{i-1,z}^j}{(\delta p)^2} + O((\delta p)^2),$$

$$(13) \quad \frac{\partial^2 V(i\delta p, z\delta q, j\delta t)}{\partial Q^2} \approx \frac{V_{i,z+1}^j - 2V_{i,z}^j + V_{i,z-1}^j}{(\delta q)^2} + O((\delta q)^2).$$

All the elements needed to approximate the partial differential equation in (4') as a finite difference equation, are now defined. Substituting (9-13) in (4'), ignoring terms of $O((\delta p)), O((\delta q)), O((\delta t))$ and $O((\delta p)^2), O((\delta q)^2)$ and rearranging the equation we obtain:

$$(14) \quad V_{i,z}^{j-1} = a_p^j V_{i-1,z}^j + a_q^j V_{i,z-1}^j + b_{p,q}^j V_{i,z}^j + c_p^j V_{i+1,z}^j + c_q^j V_{i,z+1}^j.$$

where

$$(15) \quad a_p^j = \frac{1}{2} \delta t (\sigma_p^2 - \alpha_p),$$

$$(16) \quad a_q^j = \frac{1}{2} \delta t (\sigma_q^2 - \alpha_q),$$

$$(17) \quad b_{p,q}^j = 1 + \delta t (\sigma_p^2 + \sigma_q^2 + \alpha_q + r),$$

$$(18) \quad c_p^j = \frac{1}{2} \delta t (\sigma_p^2 + \alpha_p),$$

$$(19) \quad c_q^j = \frac{1}{2} \delta t (\sigma_q^2 + \alpha_q).$$

In the implicit finite difference method, backward recursive iteration approximates the solution to the partial differential equation, V_1 , at starting time mesh point, starting from terminal time values, through the transition parameters $a_p^j, a_q^j, b_{p,q}^j, c_p^j, c_q^j$ and given known boundary conditions. In other words, the values of $V_{i+1,z}^j, V_{i,z+1}^j, V_{i,z}^j, V_{i-1,z}^j, V_{i,z-1}^j$ all depend on $V_{i,z}^{j-1}$, but in an implicit manner.

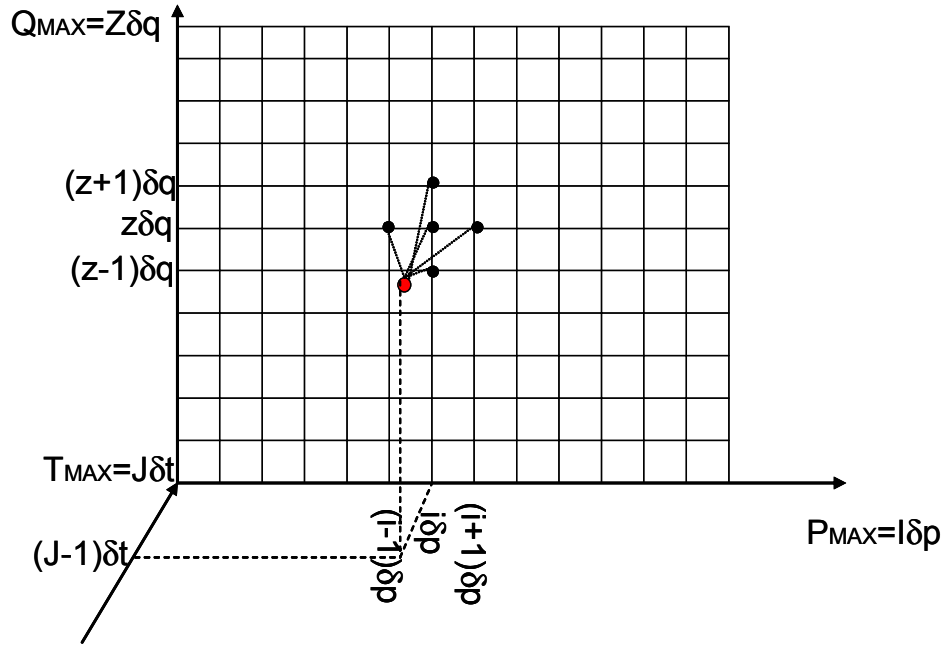


Figure 2. The relationship between option values in the implicit method

Boundary conditions are defined for a putative terminal time T_{MAX} , where for any value of price and quantity the option either will be exercised or it will expire; for $P = 0$ (for $Q = 0$), where for any value of time and quantity (price) the option is surely exercised and the fisherman has decided to irrevocably exit the fishery; and, finally, for $P=P_{MAX}$, $Q=Q_{MAX}$, where the option to exit is surely equal to zero.

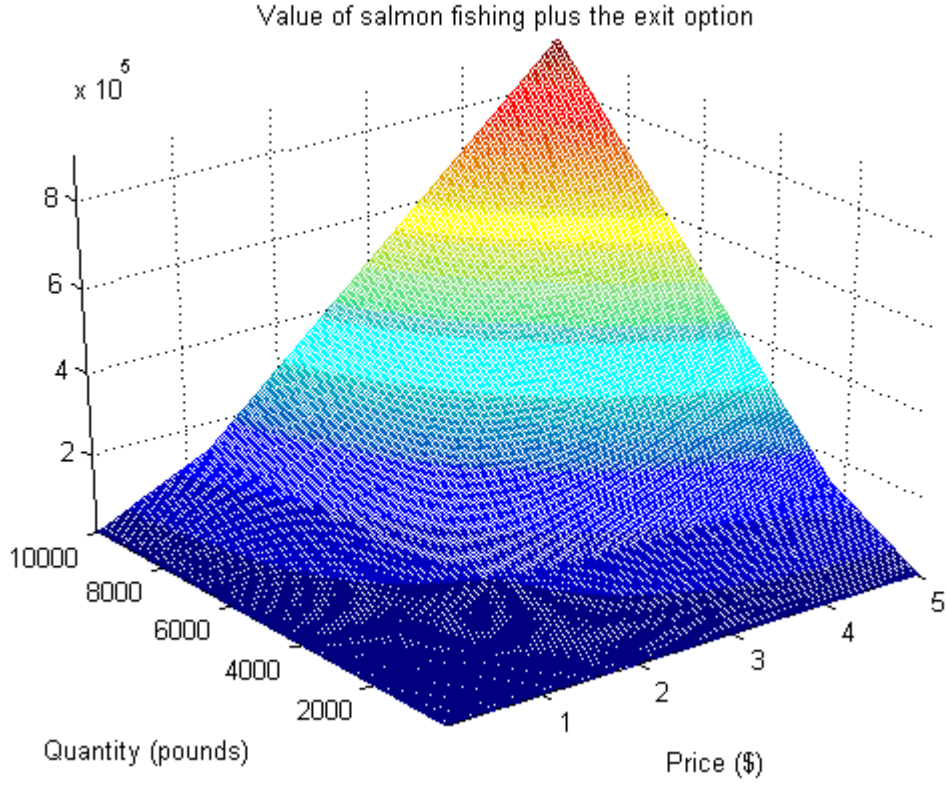


Figure 3. The function V_1 in the price-quantity space, for year 1995.

Finally, once the value of $V_{i,z}^0$ is known for the whole price-quantity space, the threshold frontier is computed as the set of points $\{P_x, Q_x\}$ where the following finite difference versions of the value matching and smooth pasting conditions hold:

$$(6') \quad V_{i,z}^0 = S,$$

$$(7') \quad \frac{V_{i+1,z}^j - V_{i-1,z}^j}{2\delta p} = 0,$$

$$(8') \quad \frac{V_{i,z+1}^j - V_{i,z-1}^j}{2\delta q} = 0.$$

The procedure was repeated for different parameters of the stochastic processes which were estimated for different years applying the procedure described in the section below.

3. Data and Parameter Estimates

Data on price and landings by boat and year were obtained from the Pacific Fisheries Information Network (PacFIN), maintained by the Pacific States Marine Fisheries Commission. Because relatively few boats have a complete time series in the data set, we estimated the parameters in equations (2-3), $\alpha_p, \sigma_p, \alpha_q, \sigma_q$, using boat average price and landings series. Maximum

likelihood estimates of the parameters were generated using rolling 6-year data horizons (e.g., 1986-1991, 1987-1992, etc.), in recognition that conditions in the fishery were changing over time and that fishermen would almost certainly have been changing their expectations as new information became available. When (2) and (3) are considered independent, the maximum likelihood estimates are available in closed form (denoting the first year of each six-year period $t=0$) as

$$(20) \quad \hat{\alpha}_p = \frac{1}{T} \sum_{t=1}^T \ln(P_t / P_{t-1})$$

$$(21) \quad \hat{\alpha}_q = \frac{1}{T} \sum_{t=1}^T \ln(Q_t / Q_{t-1})$$

$$(22) \quad \hat{\sigma}_p = \left\{ \frac{1}{T-1} \sum_{t=1}^T (\ln(P_t / P_{t-1}) - \hat{\alpha}_p)^2 \right\}^{\frac{1}{2}}$$

$$(23) \quad \hat{\sigma}_q = \left\{ \frac{1}{T-1} \sum_{t=1}^T (\ln(Q_t / Q_{t-1}) - \hat{\alpha}_q)^2 \right\}^{\frac{1}{2}}$$

Table 1: Maximum Likelihood estimates of the drift and volatility parameters for 1991-95.

YEAR	$\hat{\alpha}_p$	$\hat{\sigma}_p$	$\hat{\alpha}_q$	$\hat{\sigma}_q$
1991	0.0073	0.2051	-0.0699	0.5190
1992	-0.0346	0.1428	-0.1864	0.4907
1993	-0.0781	0.1622	-0.2038	0.4650
1994	-0.0540	0.1429	0.0597	0.2980
1995	-0.1114	0.1201	0.2014	0.3661

PacFIN data on price and landings were also used to generate proxies for annual operating costs. Specifically, a real per-trip cost of \$340 was assumed, and multiplied by the number of days of landings reported for each boat in each year. Salvage values were estimated as the difference in the median estimated boat profit among salmon boats active in a given year and the median estimated profit among salmon boats that chose to pursue other fisheries during the salmon season in that same year. Since these alternative fisheries are rather limited, the estimated salvage value using various definitions of ‘salmon boat’ (for example boats that ever exceeded 50 kg of salmon landings or that ever had more than 3 days of salmon landings per year) generally fell in the neighborhood of \$1000 per year, so we used this figure in all years for our model.

4. Model Implementation and Testing

Given the parameter estimates for the processes (2-3), we solved for the exit frontier $\{P_x, Q_x\}$ using the finite difference scheme described above. For boats reporting landings in a given year,

we then compared actual P and Q to the exit frontier, reasoning that if the model is correct and an active boat falls below the exit threshold, the boat should exit the fishery the following year and stay inactive thereafter. A boat with observed P and Q above the exit frontier in a given year should remain active in the fishery. We compared these predictions to actual behavior as determined from the PacFIN data, providing a direct test of the model's ability to predict investment behavior. Table 2 shows the results of this test for 1991-1995.

Table 2: Predicted and Observed Boat Status

	Predicted Status			% Correct
	Active	Exited		
Observed Status	Active	3044	1704	0.64
	Exited	89	222	0.71
	% Correct	0.97	0.12	0.65

Out of a total of 5059 boat-years, the model predicts a boat's status (active or exited) based on the previous year's P and Q in relation to $\{P_x, Q_x\}$ correctly 65% of the time. More informative is the breakdown of correct and incorrect predictions. Very close to all (97%) boats predicted to be active are in fact active, whereas only 12% of boats predicted to have exited the fishery had in fact done so. The dramatic failure to predict exit correctly may be due to flaws in the model that caused the exit frontier to be set too high, thus increasing the number of boats predicted to exit. However, we note that our definition of 'active' includes boats that have only temporarily suspended fishing operations, since we reserved the 'exited' status for boats that had permanently given up the fishery. Extending the model to include an intermediate status for boats that have suspended operations should have the effect of lowering the exit threshold, reducing the number of 'exit' predictions. We are currently at work on this extension. Of course, we cannot discount the possibility that the model's poor performance in predicting exit is due to a fundamentally wrong behavioral premise, namely that fishermen maximize expected profit, and anecdotal evidence from the salmon fishery indicates that for many fishermen this premise is indeed very wrong. However, our goal is a model that is useful rather than true, and the simplifying assumption of profit maximization, while obviously wrong in a strict sense, need not render the model useless. We will be in a better position to judge the model's utility after testing a version that include the fisherman's option to suspend operations.

One point worth noting is that no prediction is generated for a boat in years after which that boat does not report landings: if a boat is not active in 1992, the test described here does not apply to that boat in 1993. A more robust test would address this hole in the model's application, since

some 48% of boat-years in the data set are idle, but it is not obvious how this extension can be made without a model of behavior that depends on more than one previous period's state variables (and thus that would require state-augmented dynamic programming as a solution technique).

5. Conclusion

There are several clear limitations to the approach taken in this paper, including the use of aggregate data to explain individual behavior, the fairly arbitrary choice of 6 years as the information set on which to base expectations, and the choice of geometric Brownian motion to represent processes that probably have at least some element of mean reversion (though it must be noted that in the data set available to us neither price nor quantity shows much evidence of mean reversion). We are currently taking steps to address these weaknesses through the use of Bayesian estimators, more flexible process specifications, and models that specifically account for regulatory restrictions on re-activation of the fishing enterprise. Still, we are heartened that the relatively simple model present here has as much explanatory power as our results suggest, and conclude with the observation that such models would almost certainly perform better in more heavily-capitalized fisheries.

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